Parvatibai Chowgule College of Arts and Science

Autonomous

B.Sc. Semester End Examination, January 2022

Semester: I
Subject: Mathematics
Title: Basic Algebra (Core)
Duration: 2 hours Max. Marks: 60

Instructions: 1. All Questions are compulsory. However internal choice is available.
2. Figures to the right indicate full marks
3. Justify all your answers.

Q1. Answer **ANY THREE** of the following: **(12)**

a) Determine if $(p \to (q \vee r)) \vee (p \to q)$ is a tautology, contradiction or contingency.

b) Define Cartesian product of two sets and show that for any three sets, A, B and C, $A \times (B \cup C) = (A \times B) \cup (A \times C)$

c) Give examples of the following:

i. A binary operation which has identity but no inverse.

ii. An injective function which is not surjective.

iii. An associative binary operation which is not commutative.

iv. A surjective function which is not injective.

d) Find the $\gcd(574,483)$ and express it as $574x + 483y$.

Q2. Answer **ANY THREE** of the following: **(12)**

a) Find the inverse of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

c) Solve the equation $3x^3 - 11x^2 + 8x + 4 = 0$, given that two of its roots are equal.

d) Find the equation whose roots are the roots of $x^3 - 6x^2 + 11x - 6 = 0$ each increased by 1

Q3. Answer the following: **(12)**

a) Using rules of inference show that the following argument is valid:
If it rains, then I will take leave.
If it is hot outside, then I will go for a shower.
Either I will not take a leave or not go for a shower.
Therefore, either it does not rain or it is not hot outside.

b) (i) Let R be the relation on $\mathbb{Z}$ given by aRb if and only if $a \equiv b \pmod 5$. Show that R is an equivalence relation and hence find all the distinct equivalence classes.

OR

(ii) Let $f: A \to B$ and $g: B \to C$ be both bijective functions. Show that the composite function $g \circ f$ is bijective and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

Q4. Answer the following: **(12)**

a) Solve **ANY ONE** of the following systems of linear equations:

i) $2x - 2y + 5z + 3w = 0$
$4x - y + z + w = 0$
$3x - 2y + 3z + 4w = 0$
$x - 3y + 7z + 6w = 0$

ii) $2x + 3y + 2z = 5$
$3x - 5y + 5z = 2$
$x + 2y + z = 3$
$3x + 9y - z = 4$

b) Determine two matrices P and Q such that PAQ is in the normal form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, where $A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$

Q5. Answer **ANY TWO** of the following: **(12)**

a) Solve the reciprocal equation $6x^5 + x^4 - 43x^3 - 43x^2 + x + 6 = 0$.

b) Solve the polynomial equation $x^5 + 6x^4 - 3x^3 - 58x^2 - 54x + 36 = 0$ given that one of its roots is $\sqrt{6} - 2$.

c) Solve by Cardan's method $x^3 - 18x - 35 = 0$.

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